

corrigé niveau 2

1 Préliminaires

$$\text{Exercices 1 à 5 : } (x-5)^2 = x^2 - 10x + 25 \quad (3x+1)^2 = 9x^2 + 6x + 1 \quad (2x-1)^2 = 4x^2 - 4x + 1$$

$$(a+2b)^2 = a^2 + 4ab + 4b^2 \quad (2a-3b)^2 = 4a^2 - 12ab + 9b^2$$

Exercices 6 à 10 : $6n+7 = 6n+6+1 = 2(3n+3)+1$ donc $6n+7$ est impair car il est de la forme $2m+1$.

$$(2n-1)^2 = 4n^2 - 4n + 1 = 2(2n^2 - 2n) + 1 \text{ donc } (2n-1)^2 \text{ est impair.}$$

$$(2n)^2 - 1 = 4n^2 - 1 = 4n^2 - 2 + 1 = 2(2n^2 - 1) + 1 \text{ donc } (2n)^2 - 1 \text{ est impair.}$$

$$(n+4)^2 - (n+3)^2 = [(n+4) + (n+3)][(n+4) - (n+3)] = (2n+7) \times 1 = 2n+6+1 = 2(n+3)+1. \text{ Il est donc impair.}$$

Lorsque deux entiers se suivent, l'un est pair et l'autre est impair. Donc ici, l'un des deux (n ou $n+1$) est pair, donc $n(n+1)$ est pair.

Exercice 11 à 20 :

$$3x-9 = 3(x-3) \quad 9x^2+6x+1 = (3x+1)^2 \quad 16x^2-9 = (4x)^2-3^2 = (4x+3)(4x-3) \quad 3x^3-5x^2 = x^2(3x-5)$$

$$(2n+1)(n-7) + 3n(4n+2) = (2n+1)(n-7) + 6n(2n+1) = (2n+1)[(n-7) + 6n] = (2n+1)(7n-7) = 7(2n+1)(n-1)$$

$$x^2+2x+1 - (3x+2)(x+1) = (x+1)^2 - (3x+2)(x+1) = (x+1)[(x+1) - (3x+2)] = (x+1)(-2x-1)$$

$$(x^2-10x+25) + (x^2-5x) = (x-5)^2 + x(x-5) = (x-5)[(x-5) + x] = (x-5)(2x-5)$$

$$9(4x^2-4x+1) - 6x+3 = 3 \times 3(2x-1)^2 - 3(2x-1) = 3(2x-1)[3(2x-1) - 1] = 3(2x-1)(6x-4) = 6(2x-1)(3x-2)$$

$$(x+3)(2-x) + x^2+6x+9 = (x+3)(2-x) + (x+3)^2 = (x+3)[(2-x) + (x+3)] = 5(x+3)$$

$$(n+1)! - n \times n! = 1 \times 2 \times \dots \times n \times (n+1) - n \times 1 \times 2 \times \dots \times n = 1 \times 2 \times \dots \times n \times [(n+1) - n] = 1 \times 2 \times \dots \times n \times [1] = n!$$

2 Simplification des fractions

$$\text{Exercices 1 à 10 : } \frac{2n^3}{3n^2} = \frac{2n}{3} \quad \frac{2-2a}{3-3a} = \frac{2(1-a)}{3(1-a)} = \frac{2}{3} \quad \frac{7!}{5!} = \frac{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7}{1 \times 2 \times 3 \times 4 \times 5} = 6 \times 7 = 42 \quad \frac{7!}{5!2!} = \frac{6 \times 7}{2} = 3 \times 7 = 21$$

$$\frac{x^2-9}{x^2-3x} = \frac{(x+3)(x-3)}{x(x-3)} = \frac{x+3}{x} \quad \frac{x^2-1}{(x-1)^2} = \frac{(x+1)(x-1)}{(x-1)^2} = \frac{x+1}{x-1} \quad \frac{9x^2+6x+1}{9x+3} = \frac{(3x+1)^2}{3(3x+1)} = \frac{3x+1}{3}$$

$$\frac{16x^2-9}{4x^2-3x} = \frac{(4x+3)(4x-3)}{x(4x-3)} = \frac{4x+3}{x} \quad \frac{(n+1)(n+2)}{n^2+3n+2} = \frac{n^2+3n+2}{n^2+3n+2} = 1 \quad \frac{(n+1)!}{n!} = \frac{1 \times 2 \times \dots \times n \times (n+1)}{1 \times 2 \times \dots \times n} = n+1$$

3 Produits et quotients de fractions

$$\text{Exercices 1 à 3 : } a \times \frac{2}{3} = \frac{a}{1} \times \frac{2}{3} = \frac{2a}{3} \quad \frac{a}{\frac{3}{2}} = \frac{a}{1} \times \frac{2}{3} = \frac{2a}{3} \quad \frac{\frac{2}{3}}{\frac{1}{a}} = \frac{2}{3} \times \frac{a}{1} = \frac{2a}{3}$$

$$\text{Exercice 4 : } \frac{10^2 \times 45}{(12 \times 20)^2} = \frac{2^2 \times 5^2 \times 3^2 \times 5}{(2^2 \times 3 \times 2^2 \times 5)^2} = \frac{2^2 \times 3^2 \times 5^3}{2^8 \times 3^2 \times 5^2} = \frac{5}{2^6} = \frac{5}{64}$$

$$\text{Exercice 5 : } \frac{16000 \times 10^{-1} \times 0.02}{0.08 \times 10^3 \times (10^2)^{-4}} = \frac{16 \times 10^3 \times 10^{-1} \times 2 \times 10^{-2}}{8 \times 10^{-2} \times 10^3 \times 10^{-8}} = \frac{8 \times 4 \times 10^{3-1-2}}{8 \times 10^{-2+3-8}} = \frac{4}{10^{-7}} = 4 \times 10^7$$

$$\text{Exercice 6 : } \frac{a-1}{a+1} \times \frac{a^2-1}{a} = \frac{(a-1) \times (a-1) \times (a+1)}{(a+1) \times a} = \frac{(a-1)^2}{a}$$

$$\text{Exercice 7 : } \frac{\frac{a-1}{a+1}}{\frac{a^2-1}{a}} = \frac{a-1}{a+1} \times \frac{a}{a^2-1} = \frac{(a-1)a}{(a+1)(a+1)(a-1)} = \frac{a}{(a+1)^2}$$

Exercices 8 : $\frac{a^{2^3}}{(a^2)^3} = \frac{a^8}{a^{2 \times 3}} = \frac{a^8}{a^6} = a^2$

Exercice 9 : $\frac{\frac{0.04 \times 5 \times 10^{-2}}{1.6 \times 10^3}}{\frac{(5 \times 10^{-2})^2}{4 \times 10^5}} = \frac{0.04 \times 5 \times 10^{-2}}{1.6 \times 10^3} \times \frac{4 \times 10^5}{(5 \times 10^{-2})^2} = \frac{0.04 \times 5 \times 10^{-2} \times 4 \times 10^5}{1.6 \times 10^3 \times (5 \times 10^{-2})^2} = \frac{4 \times 10^{-2} \times 5 \times 10^{-2} \times 4 \times 10^5}{16 \times 10^{-1} \times 10^3 \times 25 \times (10^{-2})^2}$

$$\frac{\frac{0.04 \times 5 \times 10^{-2}}{1.6 \times 10^3}}{\frac{(5 \times 10^{-2})^2}{4 \times 10^5}} = \frac{16 \times 5 \times 10^{(-2)+(-2)+5}}{16 \times 25 \times 10^{(-1)+3+(-4)}} = \frac{10^1}{5 \times 10^{-2}} = \frac{2 \times 5 \times 10^2}{5} = 2 \times 10^2 = 200$$

Exercice 10 : $\frac{a^2+2a+1}{a-3} \times \frac{a^2-6a+9}{a^2-1} = \frac{(a+1)^2 \times (a-3)^2}{(a-3) \times (a+1) \times (a-1)} = \frac{(a+1) \times (a-3)}{a-1} \quad \left(= \frac{a^2-2a-3}{a-1} \right)$

4 Sommes et différences de fractions

Exercice 1 : $\frac{5}{6} - \frac{3}{10} = \frac{5}{2 \times 3} - \frac{3}{2 \times 5} = \frac{5 \times 5}{2 \times 3 \times 5} - \frac{3 \times 3}{2 \times 3 \times 5} = \frac{25-9}{2 \times 3 \times 5} = \frac{16}{2 \times 3 \times 5} = \frac{8}{15}$

Attention, si vous vous êtes arrêtés à $\frac{16}{30}$, vous n'avez pas fini le calcul.
Un calcul est terminé lorsque toutes les simplifications sont faites.

Exercice 2 à 10 : $\frac{5}{6} + 1 = \frac{5}{6} + \frac{6}{6} = \frac{11}{6}$ $\frac{3}{32} + \frac{5}{4} = \frac{3}{2^5} + \frac{5}{2^2} = \frac{3}{2^5} + \frac{5 \times 2^3}{2^5} = \frac{3+40}{32} = \frac{43}{32}$

$$\frac{3}{32} + \frac{7}{20} = \frac{3}{2^5} + \frac{7}{2^2 \times 5} = \frac{3 \times 5}{2^5 \times 5} + \frac{7 \times 2^3}{2^5 \times 5} = \frac{15+56}{160} = \frac{71}{160}$$

$$\frac{7}{120} + \frac{1}{18} = \frac{7}{2^3 \times 3 \times 5} + \frac{1}{2 \times 3^2} = \frac{7 \times 3}{2^3 \times 3^2 \times 5} + \frac{2^2 \times 5}{2^3 \times 3^2 \times 5} = \frac{21+20}{360} = \frac{41}{360}$$

$$\frac{1}{3!} - \frac{1}{4!} = \frac{4}{3! \times 4} - \frac{1}{4!} = \frac{4-1}{4!} = \frac{3}{4!} = \frac{3}{2 \times 3 \times 4} = \frac{1}{8} \quad \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = \frac{3}{6} + \frac{2}{6} + \frac{1}{6} = \frac{6}{6} = 1$$

$$\frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{3}{6} + \frac{2}{6} - \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$$

$$\frac{3}{40} - \frac{7}{60} = \frac{3}{4 \times 10} - \frac{7}{2 \times 3 \times 10} = \frac{3 \times 3}{4 \times 3 \times 10} - \frac{7 \times 2}{4 \times 3 \times 10} = \frac{9-14}{4 \times 3 \times 10} = \frac{-5}{3 \times 4 \times 10} = -\frac{1}{3 \times 4 \times 2} = -\frac{1}{24}$$

$$\frac{3}{2048} + \frac{5}{1024} = \frac{3}{2^{11}} + \frac{5}{2^{10}} = \frac{3+5 \times 2}{2^{11}} = \frac{13}{2048}$$